Abyss. J. Sci. Technol. Vol. 1, No. 1, 2016, 6-11

Abyssinia Journal of Science and Technology

Improving SIR model of Infectious Diseases by calculating at shorter time intervals

Hailay Mebrahtom*

Department of Mathematics, Aksum University, Aksum, Ethiopia

ABSTRACT

In this paper concepts related to SIR model of infectious diseases, the Mathematical model used to explain factors of the diseases vividly, and their transmission dynamics were dealt in depth. Studying the transmission dynamics of infectious diseases by making use of mathematical terms-variables was employed at different time intervals and calculations from a given set of data were carried out forward and backward in order to check the differences that may occur due to the rate differences in the time interval of the calculations. Finally, it was found out that discrepancies in the calculations got smaller and smaller when shorter time intervals were used, which in turn is meant SIR model of infectious diseases can be improved if relatively less time intervals are used in one's calculations.

Keywords: Diseases, Mathematical Model, Infectious.

INTRODUCTION

Diseases are as old as time itself. They have been there since humans were on Earth. Staying healthy does not mean never getting sick; it is beyond that. Health is about well being in mind, in body, and in community (Werner, 1993). If people are able to help themselves live healthy, they can trust each other, cooperate and work together so as to get their needs satisfied, help each other in times of adversity and plenty, learn, think better, grow and then live. Thus, we need to understand that it is our responsibility to think and do everything we can for the good of people's health. Mathematical models allow us to extrapolate from current information about the state and progress of an outbreak, to predict the future and, most importantly, to quantify the uncertainty in these predictions (Keeling & Danon, 2009).

FINDING OUT THE NEEDS

As someone who is responsible to help people stay healthy, there is a need for one to gather enough information so that one will be able to find out the needs and the concerns of the people. One has to ask questions not only to get information but also to make others ask more important questions about their felt needs, hygiene and sanitation, nutrition, and about their beliefs to healing and health. For instance, one can ask questions related to population such as how many people live in a given community, how many newborns were there this year, how many deaths were recorded this year, what were the causes of the deaths, could there be possibilities of preventing the deaths, how? Is the number of people getting larger, smaller, or remaining the same?

For how long and how often do people get sick last year, how many people did have chronic illnesses last year? What were they? Etc. Once these data are properly collected, it will be possible to develop a frame work about how to prevent and treat different diseases; and make administrative decisions accordingly.

DISEASES ARE OF TWO GROUPS

Infectious diseases - are diseases that spread from one who is infected to another who is healthy.

Non infectious diseases - are diseases that do not spread from one who is infected to another who is healthy but have other causes. To identify which diseases are infectious and which ones are not is therefore one of the important pieces of information that one needs to know for prevention and treatment matters. For instance, if we have

*Corresponding Author.
E-mail: ***hailaym@yahoo.com***

come to know that a disease is non infectious, then we do not need to give attention towards its treatment but towards its prevention as antibiotics are of no help for non infectious diseases. Now, we are leading ourselves into the discussion of infectious diseases.

MATHEMATICAL MODEL OF INFECTIOUS DISEASES

The primary goal of the study of systems is to understand their behaviors and to work for improvements accordingly. In this case, the transmission dynamics of the infectious diseases is the system and understanding its behavior in more simplified and controlled manners will be possible if identifying the contributing factors, determining their values, and predicting their future will be vivid. This is possible if the contributing factors will be represented by making use of Mathematical terms, variables (Jones, et al., 2010).

Importance: Modeling is important to draw the essential features of the system, describe them properly, predict the future spread of the disease, and to develop strategies for controlling and eradication (Murray, 2003)

Assumptions of the Model:

1. Constant Population, N
2. Everything that happens to an individual can affect others
3. Treating individuals can benefit others

Now, if some infected individuals are introduced into the large in size constant population, we ask about how the infection spreads within the population as time goes on.

The population N is divided into three disjoint classes:

Susceptible $S(t):=$ Those who are healthy and can catch the disease

Infected $I(t):=$ Those who have the disease and are capable of transmitting it

Recovered $R(t):=$ Those who are now immune

Predicting Changes: Assume that we have enough information about the values of S, I, and R this time. Can we foretell what will happen to these values as time goes on, maybe tomorrow, the day after tomorrow, after a month or after a year?

The answer to this question depends on the idea of understanding how S, I, and R change with time.

To make this idea more clear, let us assume that a new disease is spreading at a rate of I_0 new cases a day. If this rate of spread continues steadily and if there are S_0 susceptible in the beginning, then the number of susceptible we will have will decrease by I_0 each day. This means if we have S_0 susceptible today, we will have no infection today. However, we will have I_0 number of new infections and $S_0 - I_0$ susceptible left after a day that is tomorrow. The process continues this way, and Table 1 summarizes the situation that we will have in the future.

Table 1: Rate of change of the number of new infections and the number of susceptible individuals from *t = 0* to *t = n* days

Days (*t*)	**Total number of new infection (I)**	**Number of susceptible remaining (S)**
Today, $t=0$	0	S_0
Tomorrow, $t=1$	I_0	$S_0 - I_0$
The day after tomorrow, $t=2$	I_0+I_0 $=2I_0$	$(S_0-I_0)-I_0$ $=S_0-2I_0$
$t=3$	$(2I_0)+I_0$ $=3I_0$	$(S_0-2I_0)-I_0$ $=S_0-3I_0$
…..	…..	………
…..	…..	………
$t=n$	nI_0	S_0-nI_0

From this table we see that if S_0 and I_0 are the number of susceptible and the number of infections in the beginning, then after t days we will have:

$$I(n)=nI_0 \Rightarrow I(t)=tI_0$$

$$S(n)=S_0-nI_0 \Rightarrow S(t)=S_0-tI_0,$$

where $I(t)$ is the total number of infections after t days from today, and $S(t)$ is the number of susceptible remained after t days from today.

This of course, works if the rate of spread of the infection continues to remain I_0 persons a day. Now, we are going to determine that the quantities S, I and R are changing through time; that is, we will see what happens to S', I' and R'.

Rate of Recovery: Recovery is getting free of the infection through time. Someone who catches a disease will remain sick for the life time of the disease and then will recover. Suppose the infection period of the disease is j days. Then, we get the following groups of persons from the whole infected population:

1. Some who have been infected today, less than one day
2. Some who have been infected between one day and two days
3. Some who have been infected between three and four days
4. Some who have been infected for j days

Those who have been infected for j days will recover today due to the fact that the life time of the illness is j days.

Assumption: There are j groups of the same size of infected people
Therefore, in each of the j days $1/j$ of the total number of the infected people will recover. This means $R' = (i/j)I \Rightarrow R' = b\ I$, where $b = 1/j$ is called Coefficient of recovery.

Rate of Transmission: Transmission rate is about the rate of change of s during the course of the infection. S' obviously depends on both s and I because it is due to the contact between the susceptible and the infected that transmission occurs (Michael, 2001).

Since not all the infected people I can get themselves in contact with all the susceptible, let us think that only i of them do. Thus there will be $i = (i/I)I$ contacts per day per susceptible. The entire susceptible population will therefore have $(i/I)IS$ contacts each day.
Not all contacts but only a certain fraction k of them cause new infections. Hence, there will be $\mathrm{k}\,(i/I)\,\mathrm{IS}$ new infections a day.
This implies $S' = \mathrm{k}\,(i/I)\,\mathrm{IS} - a\,sI$ where $\mathrm{a} = \mathrm{k}(i/I)$ is called coefficient of transmission. The minus sign is due to the reason that S decreases on the course of the infection.

How does the group of the infected, *I* vary?

The group of the infected people I, gains members from the group of susceptible, S , and loses members to the group of recovered, R.
I…………………………gains from S : $+aSI$
I ………………………..loses to R : $-bI$
Thus, $I' = aSI - bI$

Summary

$S' = -aSI$
$I' = aSI - bI$
$R' = bI$

Observe that $S' + I +' R' = 0$. This is true because $S + I + R = N(cons\tan t)$.

NUMERICAL DESCRIPTION OF THE MODEL

To observe how these models show realities and what points they miss, let us use numerical values as elaborations. To do so, we need to have numerical values of the coefficient of transmission and the coefficient of recovery first. Consider the disease measles; an individual acquiring measles and passing it on to another individual will have an average interval of $1/25$ years and $1/25$ is approximately equal to 14 days.

Assuming steady propagation of the disease, $1/14^{th}$ of the population in the infected compartment will recover each day. Thus, $b = 1/14$.

The range of numbers used in the study of epidemics lies between 0 and 1 inclusive/exclusive. We are now free to choose $a = \dfrac{1}{100{,}000} = 0.00001.$
The rates of changes of the quantities can be now calculated if initial conditions are given. Suppose now we have the following arbitrary data of initial conditions for measles in a given community today:

$S := 50{,}000$
$I := 2{,}400$
$R := 2{,}600$

$S + I + R = 55{,}000$
$Today,\ t = 0$

These are the data we have today. Now, we are going to calculate and see what will happen to each group tomorrow, the day after tomorrow, etc.

Tomorrow, t = 1

$$R' = bI = \frac{1}{14}(2{,}400) = 171$$
$$\Rightarrow R_1 = R_0 + R' = 2{,}600 + 171 = 2771$$

$$S' = -aSI = (-0.00001 \times 50{,}000 \times 2{,}400) = -1200$$
$$\Rightarrow S_1 = S_0 + S' = 50{,}000 + -1200 = 48{,}800$$

$$I' = aSI - bI = 1200 - 171 = 1029$$
$$\Rightarrow I_1 = I_0 + I' = 2{,}400 + 1029 = 3429$$

By tomorrow, we will have the following data:

$S := 48,800$
$I := 3,429$
$R := 2,771$

$S + I + R = 55,000$
$Tomorrow,\ t = 1$

The iteration continues this way until all the infected get recovered (Table 2).

In order to see to what extent the model reflects realities and minimizes errors, let us check it backwards.

Table 2: Values of S, I, R, S', I', and R' calculated forward from *t = 0* to *t = 3* days

t	S	I	R	S'	I'	R'
0	50,000	2,400	2,600	-1200	1029	171
1	48,800	3,429	2,771	-1673.352	1428.352	244.93
2	47,126.648	4,857.422	3,015.93	-2289	1942	346.96
3	44,837.648	6,799.422	3,361.96	-	-	-

These data of tomorrow will be the initial data for the day after tomorrow, and the iterations will continue accordingly.

<u>After tomorrow, t = 2</u>

$$R' = bI = \frac{1}{14}(3,429) = 244.93$$

$$\Rightarrow R_2 = R_1 + R' = 2,771 + 244.93 = 3015.93$$

$$S' = -aSI = (-0.00001 \times 48,800 \times 3,429)$$
$$= -1673.352$$

$$\Rightarrow S_2 = S_1 + S' = 48,800 + 1673.352$$
$$= 47,126.648$$

$$I' = aSI - bI = 1673.352 - 244.93 = 1428.422$$

$$\Rightarrow I_2 = I_1 + I' = 3,429 + 1428.422 = 4857.422$$

By the day after tomorrow, we will have:

$S := 47,126.648$
$I := 4,857.422$
$R := 3,015.93$

$S + I + R = 55,000$
$Aftertomorrow,\ t = 2$

<u>The next day, t =3</u>

$$R' = bI = \frac{1}{14}(4,857.422) = 346.96$$

$$\Rightarrow R_3 = R_2 + R' = 3,015 + 346.96 = 3,361.96$$

$$S' = -aSI = (-0.00001 \times 47,126.648 \times 4,857.422)$$
$$= -2289$$

$$\Rightarrow S_3 = S_2 + S' = 47,126.648 + -2289$$
$$= 44,837.648$$

$$I' = aSI - bI = 2289 - 346.96 = 1942$$

$$\Rightarrow I_3 = I_2 + I' = 4,857.422 + 1942 = 6,799.422$$

Now, we are going to calculate all the values back from tomorrow to today to help us observe if we will get the values that we started our calculations with.

<u>Tomorrow, t = 1</u>

$S := 48,800$
$I := 3429$
$R := 2771$

$S' := 48,800$
$I' := 3429$
$R' := 2771$

WTS: coming back to today and check if results will agree.

<u>Today, t = 0</u>

$$R' = 244.93$$

$$\Rightarrow R_0 = R_1 - R' = 2771 - 244.93 = 2526.07$$

$$S' = -1673.352$$

$$\Rightarrow S_0 = S_1 - S' = 48,800 - (-1673.352) = 50,473.352$$

$$I' = 1428.352$$

$$\Rightarrow I_0 = I_1 - I' = 3429 - 1428.352 = 2000.648$$

Path: Calculating from tomorrow to today

Expected values to be obtained:

$S := 50,000$
$I := 2400$
$R := 2600$

Obtained values:

$S := 50,473.352$
$I := 2000.648$
$R := 2526.07$

Differences

$\mathrm{a}S = 473.352$

$\mathrm{a}I = -399.352$

$\mathrm{a}R = -73.93$

Where have the differences come from?

It is reasonable for us to think that these differences have been the result of either flaw in the model or differences in the rates in the time interval we calculated the numbers. And to determine which of these is the cause of the differences in the results when we go forward and backward, let us try calculating at shorter intervals than we did and see if there will be decrements in the gaps of the results. Since the number of susceptible, the number of infected, and the number of recovered are all functions of time, it will be wise to think that the results we will get by calculating at a shorter time interval than we did will bring about some improvements in correcting the discrepancies of the results. Until this time, we have been calculating the numbers at a time interval $\mathrm{a}t = 1$, i.e., once a day or once in every twenty four hours. However, the rate of transmission and the rate of recovery of the disease cannot be exactly the same each and every day. Thus it will be better to check twice, thrice, four times, and even ten or hundred times a day to get relatively better results that maybe close to exact values.
Now, let us again consider the initial data we considered in the beginning and calculate the rates of changes at a lesser interval of time $(\mathrm{a}\,t = 0.1)$ than before and check the changes.

$S_0 = 50{,}000$ $\quad S'_0 = -1200$

$I_0 = 2{,}400$ $\quad I'_0 = 1029$

$R_0 = 2{,}600$ $\quad R'_0 = 171$

$\mathrm{a}t = 0.1$

$\mathrm{a}t = 0.1$ Means we check the rtes of changes every six minutes a day.

$\mathrm{a}S = S'.\mathrm{a}t = -1200(0.1) = -120$

$\mathrm{a}I = I'.\mathrm{a}t = 1029(0.1) = 102.9$

$\mathrm{a}R = R'.\mathrm{a}t = 171(0.1) = 17.1$

$\Rightarrow S_1(t = 0.1) = S_0 + \mathrm{a}S = 50{,}000 - 120 = 49{,}880$

$I_1(t = 0.1) = I_0 + \mathrm{a}I = 2400 + 102.9 = 2502.9$

$R_1(t = 0.1) = R_0 + \mathrm{a}I = 2600 + 102.9 = 2702.9$

$S'_1 = -aS_1I_1 = (-0.00001)(49880)(2502.9)$
$= -1248.45$

$R'_1 = bI_1 = \frac{1}{14}(2502.9) = 178.78$

$I'_1 = aS_1I_1 - bI_1 = 1248.45 - 178.78 = 1069.67$

<u>Now, t = 0</u>

$S := 50{,}000$ $\quad S'_0 = -1200$

$I := 2400$ $\quad I'_0 = 1029$

$R := 2600$ $\quad R'_0 = 171$

<u>After t = 0.1 hour (6 minutes)</u>

$S := 49880$ $\quad S' := -1248.45$

$I := 2502.9$ $\quad I' := 1069.67$

$R := 2702.9$ $\quad R' := 178.78$

Path: Calculating from t = 0.1 to t = 0

1) **Expected values to be obtained**:

$S := 50{,}000$

$I := 2400$

$R := 2600$

We now use the data at t = 0.1 as our initial values and calculate the values back to t = 0.

$\mathrm{a}S = S'.\mathrm{a}t = -1248.45(0.1) = -124.845$

$\mathrm{a}I = I'.\mathrm{a}t = 1069.67(0.1) = 106.967$

$\mathrm{a}R = R'.\mathrm{a}t = 178.78(0.1) = 17.878$

$\Rightarrow S_0 = S_1 - \mathrm{a}S = 49880 - (-124.845) = 50004.845$

$I_0 = I_1 - \mathrm{a}I = 2502.9 - 106.967 = 2395.933$

$R_0 = R_1 - \mathrm{a}R = 2702.9 - 17.878 = 2685.022$

2) **Obtained values**:

$S := 50004.845$

$I := 2395.933$

$R := 2685.022$

3) **Differences**

$\mathrm{a}S = 50004.845 - 50{,}000 = 4.845$

$\mathrm{a}I = 2395.933 - 2400 = -4.067$

$\mathrm{a}R = 2685.022 - 2600 = 85.022$

<u>Differences of going forward and backward at t = 1</u>

$\mathrm{a}S = 473.352$

$\mathrm{a}I = -399.352$

$\mathrm{a}R = -73.93$

<u>Differences of going forward and backward at t = 0.1</u>

$$\text{a}S = 4.845$$

$$\text{a}I = -4.067$$

$$\text{a}R = 85.022$$

From the present study, it can be concluded that the discrepancies are so small when t = 0.1 that the results are almost the same when we go forward and when we go back ward. Therefore, it is possible to come to exact values of the S-I-R model by calculating the values at very shorter intervals of time due to the fact that the rates of changes of the infectious diseases cannot be the same over the whole course of a day.

REFERENCES

Jones, D. S., Plank, M. & Sleeman, B. D. (2010). Differential equations and mathematical biology. CRC press, U.S.A.

Keeling, J.M. & Danon, L. (2009). Mathematical modeling of infectious diseases. *British Medical Bulletin*, *92(1)*, 33-42.

Michael, A. D. (2001). An introduction to mathematical modelling. Amazon publishers.

Murray, J. D. (2003). Mathematical biology. Springer-Verlag Berlin Heidelberg.

Werner, D. (1993). Where there is no doctor - A village health care Handbook, Macmillan Education Ltd, London.